Closing Tues: Sup. 1-3, Sup. 4
Closing ~~Thes~~ Tues: Sup. 5

Entry Task: (Try on your own)

Sup. 5 / Problem 3:

A car and a bus start from the same place. Use *C(t)* and *B(t)* to denote the distance (in miles) they each traveled, respectively, after *t* min.

Part (b) TRANSLATE TO FUNCTIONAL NOTATION:

"The average trip speed of the car at 3 minutes is the same as the average speed of the bus over the 2-minute interval starting at time *t*."

BOTH CORRECT

CAR

$$\text{AVERAGE TRIP SPEED} = \frac{C(t) - C(0)}{t - 0} = \text{OVERALL RATE}$$

SO

$$\text{AVERAGE TRIP SPEED At } t=3 = \frac{C(3) - C(0)}{3 - 0}$$

BUS

$$\text{AVERAGE SPEED FOR BUS} = \frac{B(b) - B(a)}{b - a}$$

$$\text{START} = a = t$$
$$\text{END} = b = t+2$$

So

$$\text{AVERAGE SPEED FROM } t \text{ TO } t+2 = \frac{B(t+2) - B(t)}{t+2 - t}$$

$$= \frac{B(t+2) - B(t)}{2}$$

IF THEY ARE THE SAME (EQUAL), THEN

$$\frac{C(3) - C(0)}{3} = \frac{B(t+2) - B(t)}{2}$$

NOTE: SINCE C(t) = DIST TRAVELED SINCE THE START WE KNOW C(0) = 0, SO IT IS ALSO CORRECT TO WRITE

$$\frac{C(3)}{3} = \frac{B(t+2) - B(t)}{2}$$

Another HW Problem

Sup. 5 / Problem 4: The graph of y = f(x) is given. How would you answer the questions below? Write your answer in words.

SLOPE AT 2
SLOPE AT 3
SLOPE AT 5
SLOPE AT 8
UP 2→
f(x)

Part (d):
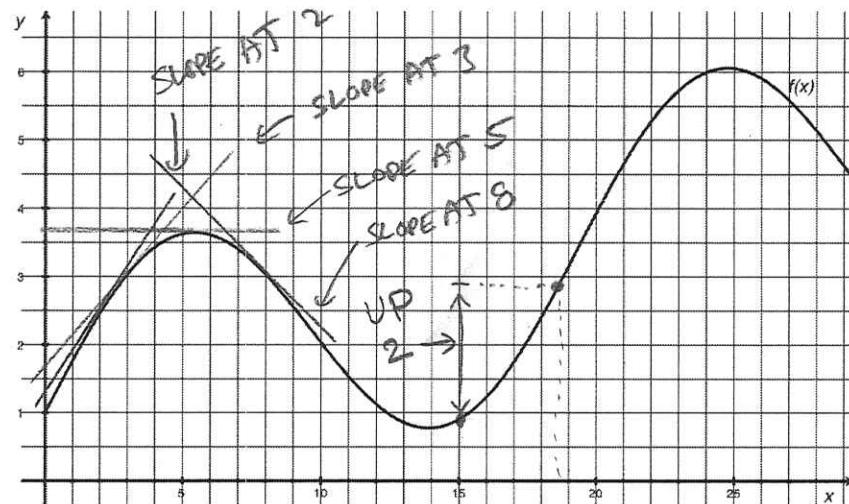
Find x such that $f(x) - f(15) = 2$.

"CHANGE IN HEIGHT FROM 15 TO x"

• START AT 15
• WANT HEIGHT TO CHANGE BY 2 (UP 2)
• FIND END OF INTERVAL

ABOUT $x \approx 18.7$

Part (h): As x takes on every value from x = 2 to x = 8, which best describes the values of $\dfrac{f(x+0.1)-f(x)}{0.1}$?

   i) They increase.

   ii) They increase, then decrease.

   iii) They decrease.

   iv) They decrease, then increase.

"SLOPE OF SECANT FROM x TO x+0.1"
LOOKS LIKE A TANGENT!

FROM GRAPH WE CAN SEE THESE
SLOPE GO DOWN AT x GOES FROM 2 TO 8.

Sup. 6: Revenue, Cost, Profit

Story: You own a cupcake business.
- You charge $1.50 per cupcake.
- Each day, your rent/utilities are $120 and your salaries for employees are $180.
- It costs you $0.50 to produce each cupcake.

We will answer the questions:
a) How many cupcakes should you produce and sell to "break even"?

b) How many should you produce and sell to make a profit of $100?

Key Concepts we will introduce:
1. Total Revenue (TR)
2. Total Costs (TC)
3. Fixed Costs (FC)
4. Variable Costs (VC)

Note:

We will use q, or x, for the number of items produce or sold (quantity).

1. Total Revenue (TR)

total amount of money you receive from selling q items

$TR(0) = 0$ ← ALWAYS FOR TR

$TR(1) = \$1.^{50}$

$TR(2) = \$3.^{00}$

$TR(3) = \$4.^{50}$

$\vdots \qquad \vdots$

$TR(10) = \$15.^{00}$

$\boxed{TR(q) = PRICE \cdot QUANTITY}$

IN THIS CASE

$\boxed{TR(q) = 1.50 \cdot q}$ ← A LINE!

slope

2. Total Cost (TC)

total amount of money you pay to produce q items.

[A] FIXED COST (FC) = COSTS YOU PAY TO BE OPEN (PRODUCE 0 ITEMS) ex: rent, utilities, salaries,...

$FC = 120 + 180 = \$300$ ← y-intercept!

[B] VARIABLE COST (VC) = COSTS THAT DEPENDS ON NUMBER OF ITEMS PRODUCED.

Ex $VC(0) = 0$ ← ALWAYS FOR VC!

$VC(1) = 0.^{50}$

$VC(2) = 1.^{00}$

$VC(3) = 1.^{50}$

$\vdots \qquad \vdots$

$VC(10) = 5.^{00}$

$VC(q) = \binom{COST\ PER}{ITEM} \times (QUANTITY)$

$VC(q) = 0.5q$

$\boxed{TC(q) = FC + VC(q) = 300 + 0.5q}$

SLOPE

① BREAK EVEN?

PROFIT = 0 ←

REVENUE = COST

$q = 300$
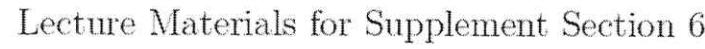
② WHEN IS PROFIT = "100?

VERT. GAP = 100

FROM GRAPH THIS
HAPPENS WHEN $q = 400$

TC(0) = FC

Fixed Costs →

Lecture Materials for Supplement Section 6

slope = 1.^{50}

TR

"BREAK EVEN"

VERTICAL GAP = PROFIT = 100

TC
slope = 0.50

VC
slope = 0.50

dollars

quantity (Things)
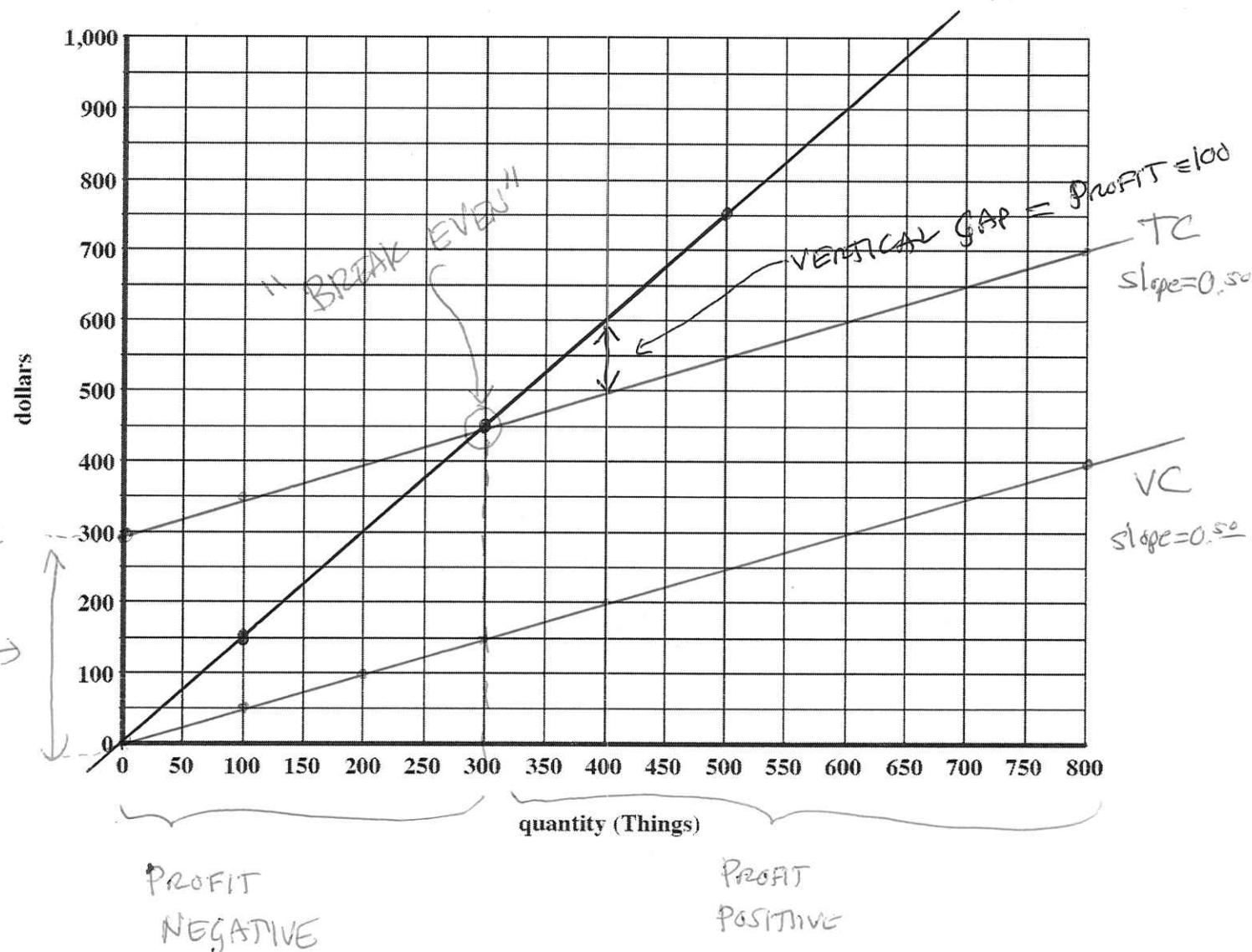
PROFIT
NEGATIVE

PROFIT
POSITIVE

TR(q) = (Price/item)*(items sold)

FC = Fixed Costs
VC(q) = (Cost/item)*(items made) = production cost for q items
TC(q) = FC + VC(q)

P(q) = Profit to make/sell q items = TR(q) – TC(q)
     = Vert. gap between TR & TC

Supp. 7: Marginal Revenue, Marg. Cost, Marg. Profit

MR(q) = additional revenue from selling the <u>next</u> item

MC(q) = additional cost in producing the <u>next</u> item

MP(q) = additional profit in prod/selling <u>next</u> item

Note: MP(q) = MR(q) – MC(q)

Q: In the cupcake scenario, what is the MR, MC, and MP at q = 400 items?

$MR(400) = \$1.50$

$MC(400) = \$0.50$

$MP(400) = \$1.00$

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IF WE HAVE PRODUCED AND SOLD 400 CUPCAKES, THEN WE PRODUCE AND SELL 1 MORE (THE 401st CUPCAKE)

IT WILL BRING IN $\$1.50$ in revenue
IT WILL COST $\$0.50$.
SO CHANGE IN PROFIT WILL BE $\$1.00$

Observation:

If MR(q) > MC(q), then MP(q) is positive and profit will go up if we sell one more item.

If MR(q) < MC(q), then MP(q) is negative and profit will go down if we sell one more item.

FOR THE CUPCAKE EXAMPLE $MR(q) = \$1.\underline{50}$, $MC(q) = \$0.\underline{50}$

ARE CONSTANTS, BUT IN MANY BUSINESS EXAMPLES THEY ARE NOT CONSTANT. SO LET'S DISCUSS A GRAPH THAT IS NOT A STRAIGHT LINE...
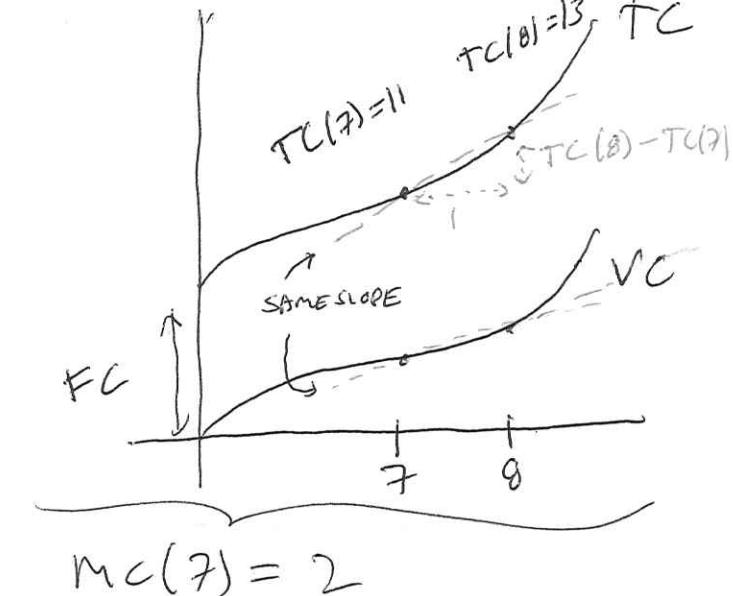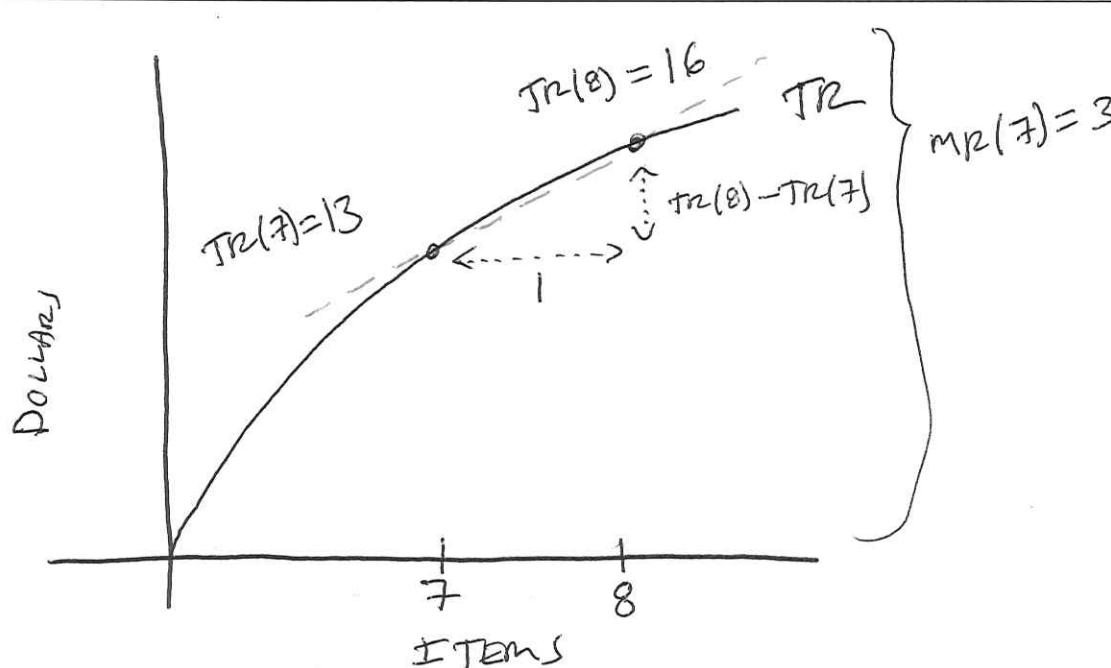
Translating MR/MC to
Graphs/Notation:

$$MR(q) = TR(q+1) - TR(q) = \text{"change in height on TR"}$$
$$= \frac{TR(q+1) - TR(q)}{(q+1) - (q)} = \text{"slope of secant to TR"}$$

$$MC(q) = TC(q+1) - TC(q) = \text{"change in height on TC"}$$
$$= \frac{TC(q+1) - TC(q)}{(q+1) - (q)} = \text{"slope of secant to TC"}$$

Computation Exercises:
Use lines and slopes to estimate:

1. What is marginal cost at 6 hundred Blinkos?

600 items $\Longleftrightarrow$ $q = 6$ hundred items

601 items $\Longleftrightarrow$ $q = 6.01$ hundred items

$$mc(6) = \frac{TC(6.01) - TC(6)}{0.01} \qquad \frac{\text{hundreds of \$}}{\text{hundreds of items}}$$

= "slope of secant from 6 to 6.01"

LOOKS LIKE A TANGENT!

DRAW LINE! TWO PTS: $(6, 30)$, $(38, 60)$

$$SLOPE \approx \frac{60 - 30}{38 - 6} = \frac{30}{32} \approx 0.9375$$

$$\boxed{mc(6) \approx \$0.\underline{94}}$$

AROUND $\$1.^{00}$ to produce to 601st item.

2. At what quantity is marginal cost equal to 3 dollars/item?

GIVEN A SLOPE!

DRAW A REFERENCE LINE!

SLOPE = 3 $(0,0)$, $(10, 30)$
                      $(20, 60)$ . .
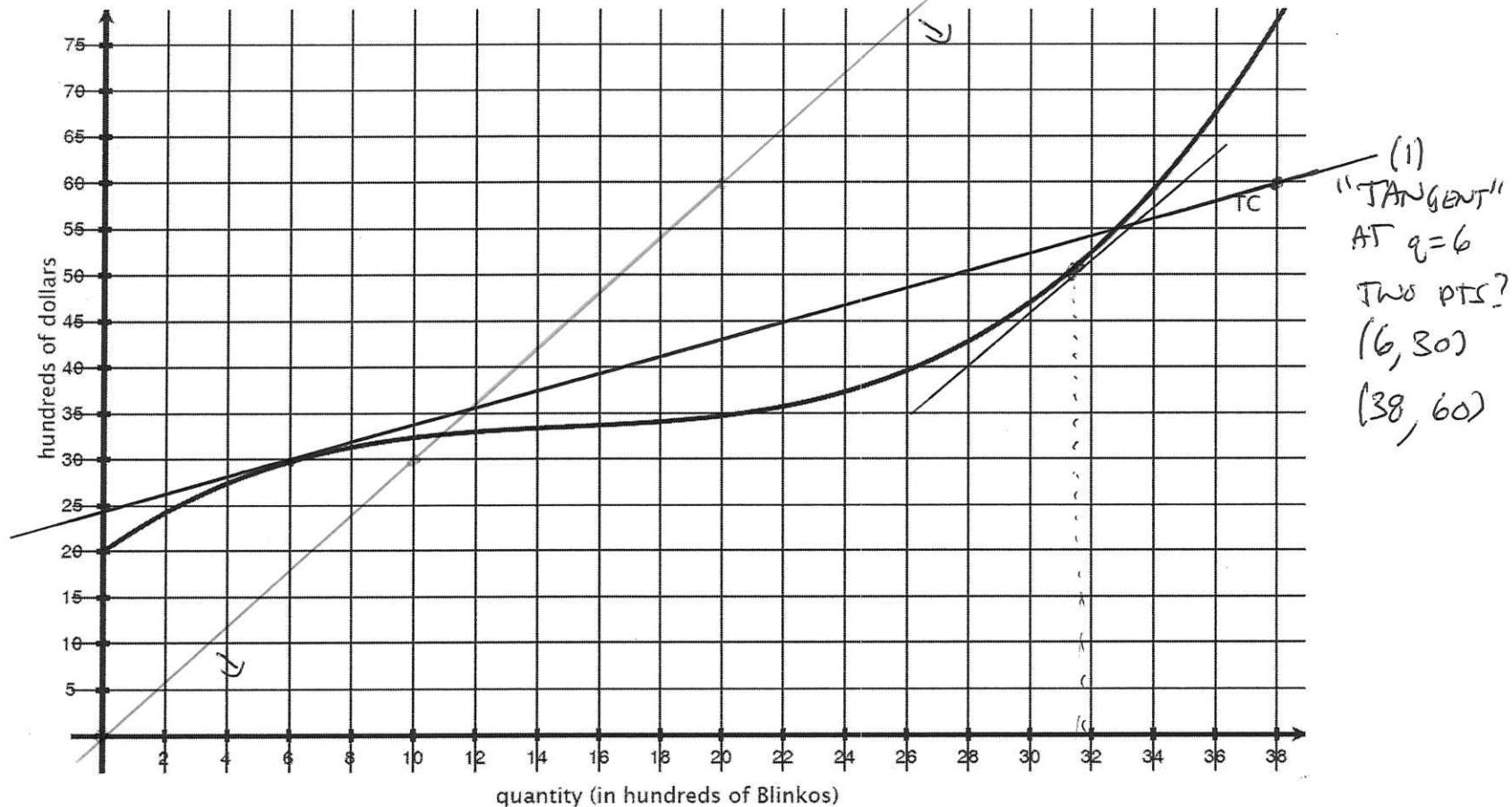
- SLIDE PARALLEL UNTIL
  IT LOOKS LIKE A TANGENT.

$q \approx 31.50$ hundred items

So mc at
3150 items
is about $\$3/item$

Example: Blinkos (lecture pack)
Here the units are in hundreds,
 1 Blinko = 0.01 hundred Blinkos

So $MR(q) = \dfrac{TR(q+0.01) - TR(q)}{q+0.01 - q} = \dfrac{\$}{item}$

(2) slope $= \$3/item$

(1)
"TANGENT"
AT $q=6$
TWO PTS?
$(6, 30)$
$(38, 60)$

quantity (in hundreds of Blinkos)

hundreds of dollars

TC

Suppose we sell Blinkos at a price
of $2.00 per Blinko.

1. Draw the TR graph
2. How many quantities should
   you sell in order to maximize
   profit?

TR = diagonal line with slope 2

$(0,0)$ $(1,2)$ ... $(10,20)$, $(30,60)$ ...

" LARGEST VERTICAL GAP WHEN $TR > TC$ "

OCCURS WHEN " SLOPES MATCH "

MAX PROFIT OCCURS WHEN $q \approx 29$ hundred items
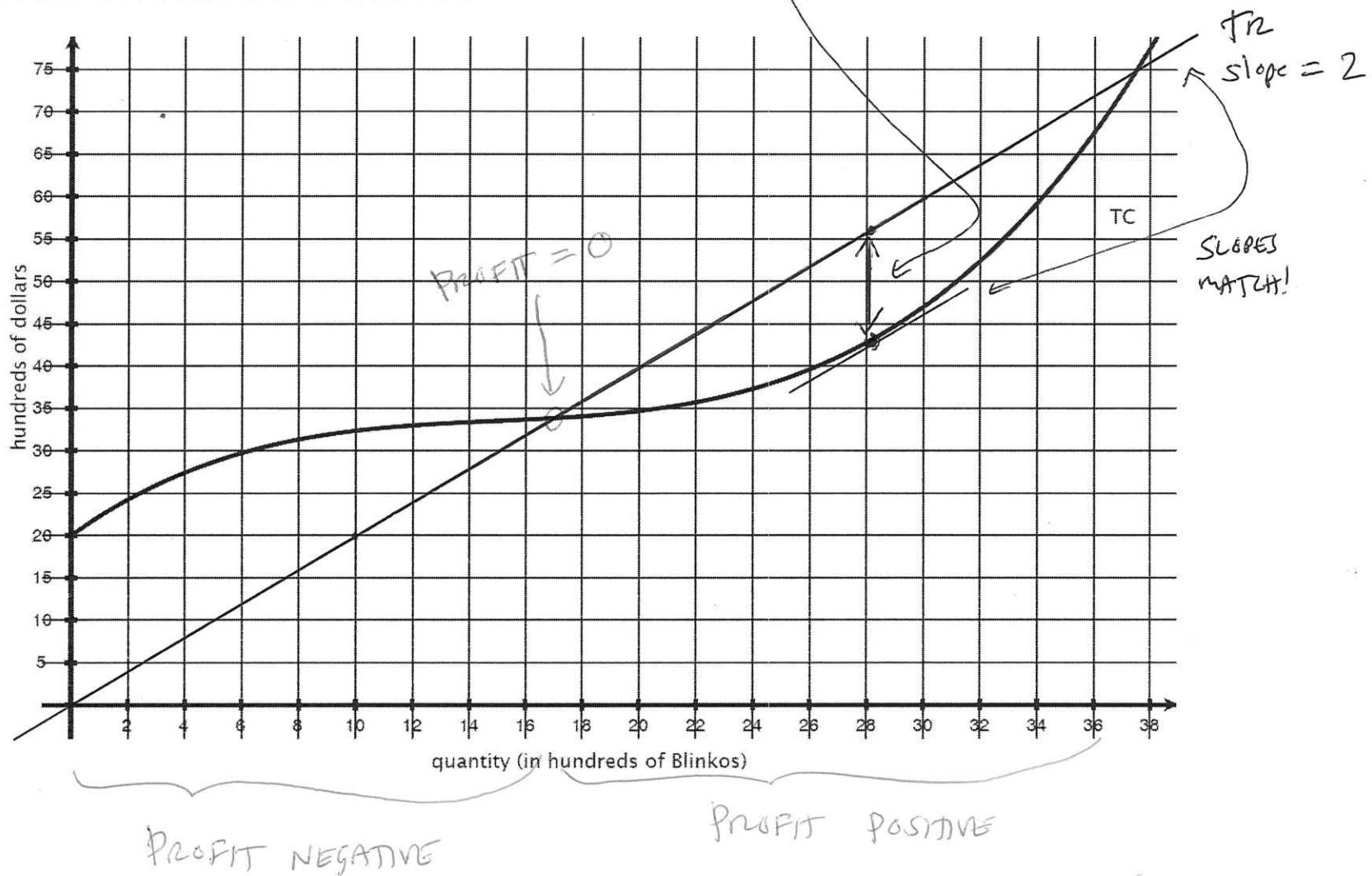
AND MAX PROFIT $\approx 56 - 43$ = 13 hundred dollars

Example: Blinkos (lecture pack)
Here the units are in hundreds,
 1 Blinko = 0.01 hundred Blinkos

MAX PROFIT = "LARGEST VERTICAL
GAP WHEN TR IS ABOVE TC"

OCCURS WHEN SLOPES
MATCH!

TR
slope = 2

TC

SLOPES MATCH!

PROFIT = 0

PROFIT NEGATIVE

PROFIT POSITIVE

quantity (in hundreds of Blinkos)

hundreds of dollars

OCCURS AROUND   28   or   29 hundred items

VERTICAL GAP ≈ 56 - 43 = 13 hundred dollars

Max Profit Methods Summarized

Method 1: Largest Gap

Find the largest vertical gap when TR is above TC.

    Pro = quick

    Con = hard to be accurate

Method 2: Match Slopes

Find where TR and TC have the same tangent slopes. Best way if given TR and TC graphs!

Method 3: MR = MC

Find where it switches from MR > MC to MR < MC

Note: MR>MC→profit increasing

             MR<MC→profit decreasing

    Pro = precise (algebra)

    Con = need MR/MC formulas

Sup. 6-7 HW / Problem 2 Hints:
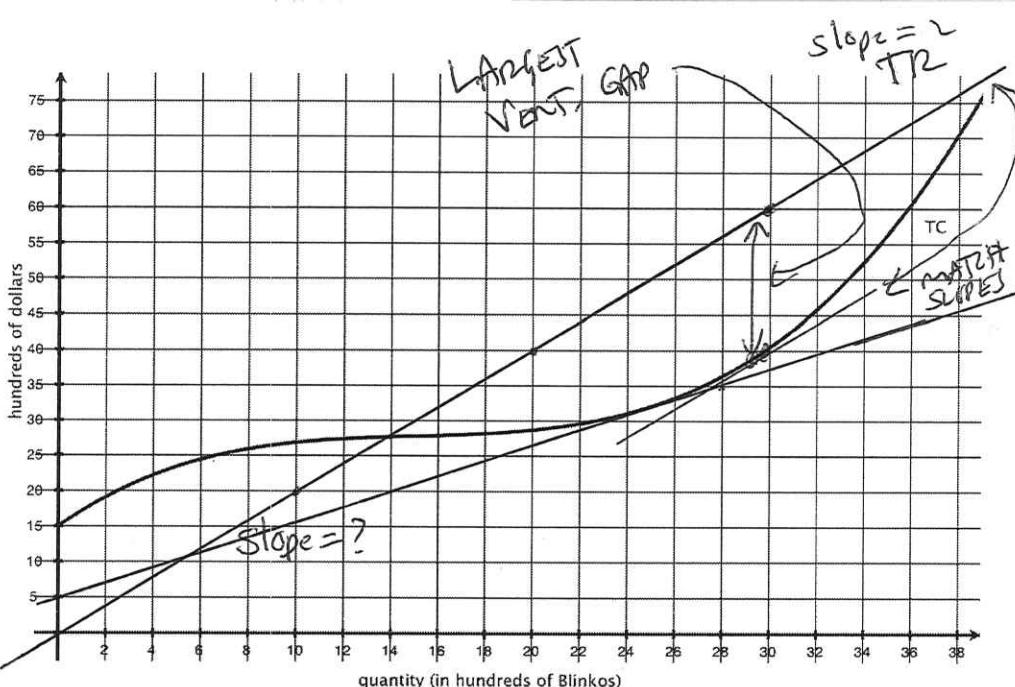
Do you know how to answer these...

(a) What is TC(38) and VC(38)?

$TC(38) = 70$ hundred dollars

SINCE $FC = 15$ hundred dollars

$VC(38) = 70 - 15 = 55$ hundred dollars

LARGEST VERT. GAP

slope = 2

TR

TC

← MATCH SLOPES

Slope = ?

hundreds of dollars

quantity (in hundreds of Blinkos)

(b) How would you estimate MC at 24 hundred items?

≈ SLOPE OF TANGENT AT $q = 24$

GET TWO PTS!

$(14, 20)$ $(28, 35)$

$$\frac{35 - 20}{28 - 14} = \frac{15}{14} \approx 1.07/\text{item}$$

The 2,401st item will cost about \$1.07 to produce

(c) If the items sell for \$2 per item what is the maximum profit?

DRAW TR!

LARGEST VERTICAL GAP (MATCH SLOPES!)

• OCCURS AROUND $q = 29$ or 30

• GAP ≈ $60 - 40 ≈$ 20 hundred dollars

≈ \$2,000

Sup. 6-7 HW / Problem 3 Hints:

Do you know how to answer these...

(a) What is the functional notation for the MC(q)? (Note: q is in thousands of items!)
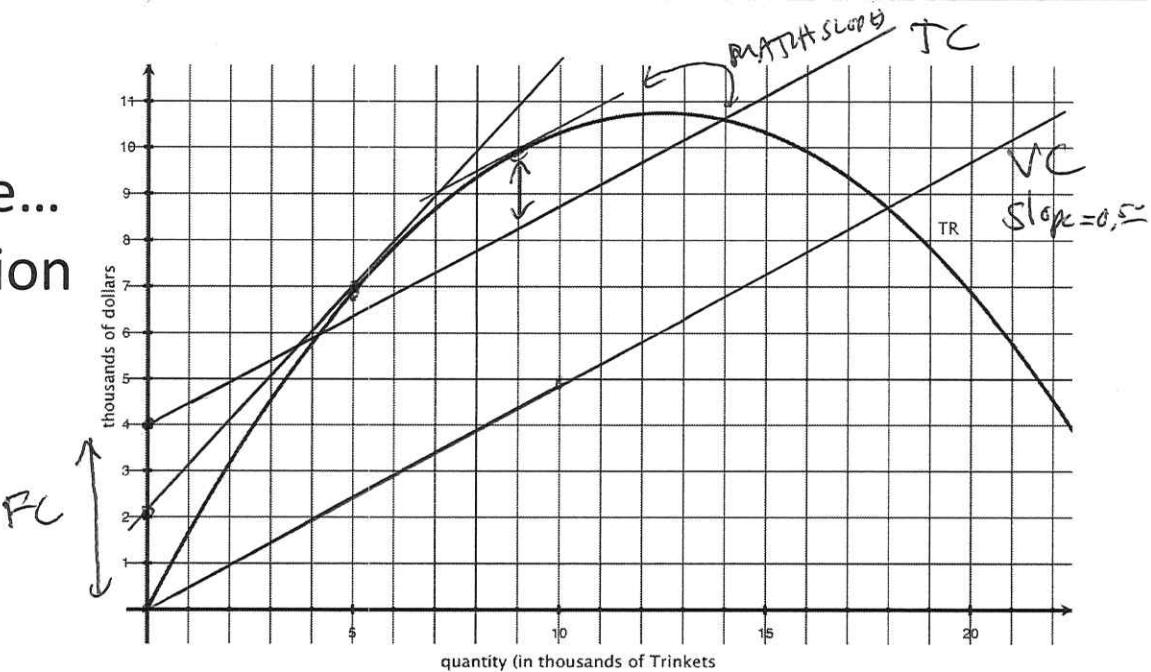
$$\frac{TC(q + 0.001) - TC(q)}{0.001} \underset{\longleftarrow}{\text{one item}}$$

FC

MATCH SLOPE   TC

VC   Slope = 0.5

TR

thousands of dollars

quantity (in thousands of Trinkets)

(b) How would you compute marginal revenue at 5 thousand items?

DRAW A "TANGENT LINE" AT q=5.
ESTIMATE SLOPE. TWO PTS: (0,2) (9,11)

$$SLOPE \approx \frac{11-2}{9-0} = \frac{9}{9} = \$1.\overset{00}{-}/item$$

(c) If it costs $0.50 to produce every item and the fixed costs are $4000, then what is maximum profit?

DRAW TC!   Slope = 0.50

LARGEST VERTICAL GAP (MATCH SLOPES)
OCCURS AROUND q = 9 thousand items

GAP ≈ 10 - 8.2 = 1.8 thousand $.

MAX. PROFIT ≈ $1,800